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THE NUMERICAL SOLUTION OF THE HEAT CONDUCTION EQUATION OCCURING IN THE THEORY OF THERMAL EXPLOSIONS

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7 NOVEMBER 1956

U. S. NAVAL ORDNANCE LABORATORY
WHITE OAK, MARYLAND

THE NUMERICAL SOLUTION OF THE HEAT CONDUCTION EQUATION
OCCURRING IN THE THEORY OF THERMAL EXPLOSIONS

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ABSTRACT: The non-linear heat conduction equation which
arises in the theory of thermal explosions has been solved
by numerical integration for the semi-infinite slab and
solid cylinder, and sphere geometries without restrictive
approximations. Within the range of practical interest,
the values for the critical conditions of inflammability
obtained are very similar to those found by previous
investigators who solved the equation both analytically
and numerically, under restrictive approximations. Hence,
within this range, the new more exact solution justifies
the earlier approximate result of Frank-Kamenetsky (1).
Application of the results to engineering problems is
given; in particular to the design of large rocket grains
where self-heating may result in "spontaneous" ignition
during manufacture or storage.

EXPLOSIVES RESEARCH DEPARTMENT
U. S. NAVAL ORDNANCE LABORATORY
White Oak, Silver Spring, Maryland

The computations reported herein are believed to be accurate,
complete and applicable to practical systems within the limits
set by the mathematical model. For example, the effect of
liberated gases, which would lower the thermal conductivity
if trapped in the solid, is ignored in the theoretical treatment.

The required constants are not available for the majority of
propellants used today. While the quoted example, cordite,
gives no cause for concern we do not have confidence that this
is true for all propellants. The possibility of spontaneous
combustion during the manufacture and storage of very large
rocket grains should be evaluated very carefully in terms of
these computations.

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12-56 and NO 800-667/76004/01040.

This report is the responsibility of the authors and the
Fuels and Propellants Division of the Naval Ordnance Laboratory.

W. W. WILBOURNE
Captain, USN
Commander

J. E. ABLARD
By direction

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INTRODUCTION

Let a combustible undergo an exothermic chemical reaction with heat loss to the walls of the containing vessel where this loss is assumed to take place via a conductive process inside the combustible volume. The criterion for thermal explosion is that the energy liberated during the reaction must be greater than that lost through the surfaces. The condition when the heat loss exactly compensates energy production in the combustible is described by the steady temperature state.

The steady-state heat conduction equation can be written as

$$\lambda\left(\frac{\partial^2 T}{\partial x^2} + \frac{m}{x}\frac{\partial T}{\partial x}\right) = -\rho Q Z \exp\left(-\frac{E}{RT}\right) ; \quad |x| < b \tag{1}$$

where T is the combustible temperature, x the space coordinate, λ the thermal conductivity, ρ the density, Q the heat of reaction, Z the frequency factor, E the activation energy, R the gas constant, and b the half-thickness of the combustible. $m = 0$, 1, and 2 for the slab, cylinder, and sphere geometries respectively. It is assumed that the reaction is first order. The solution of Eq. (1) for the semi-infinite slab immersed in an isothermal bath at temperature T_s, under the assumption that $T - T_s \ll T_s$ was first given by Frank-Kamenetsky (1). With the same assumption, Chambré (2) showed that the solution of Eq. (1) for solid cylindrical and spherical geometries can be obtained in terms of known functions. Also using the same assumption Enig, (3) presented solutions for the case of the hollow cylinder and showed that from these, the slab and solid cylinder solutions can be deduced as special cases. In the work of the above investigators a parameter δ defined as

$$\delta = \frac{\rho Q Z E}{\lambda R T_s^2} b^2 \exp\left(-\frac{E}{RT_s}\right) \tag{2}$$

relates the surface temperature to the half-thickness. For each particular geometry, the steady-state is possible only if the value of δ is less than or equal to a certain critical value. For values in excess of the critical one, explosion will occur. Hence for any surface temperature (half-thickness) there is a critical thickness (surface temperature) which if exceeded will lead to explosion.

The solution of Eq. (1) by numerical integration for the semi-infinite slab and solid cylinder, and sphere geometries, without restrictive approximations has been carried out; the critical conditions of explosion obtained have been compared with those found by Frank-Kamenetzky and Chambré.

MATHEMATICAL MODEL

Introducing dimensionless variables, Eq. (1) can be written

$$\frac{\partial^2 \theta}{\partial \eta^2} + \frac{m}{\eta} \frac{\partial \theta}{\partial \eta} = -\exp\left(-\frac{1}{\theta}\right) \quad ; \quad |\eta| < \eta_s \qquad (3)$$

where

$$\eta = \left(\frac{\rho Q Z R}{\lambda E}\right)^{\frac{1}{2}} x \quad , \quad \theta = \frac{RT}{E} \, , \, \eta_s = \eta(b) \qquad (4)$$

The boundary conditions are

$$\begin{cases} \theta = \theta_0 \\ \eta = 0 \end{cases} \qquad \begin{cases} \theta = \theta_s \\ \eta = \eta_s \end{cases} \qquad (5)$$

where θ_s and η_s are the dimensionless surface temperature and half-thickness respectively.

One method of solving Eq. (3) is to write it as a finite difference equation. Choose an arbitrary value of η_s. Assume a value of θ_0 and solve the difference equation for θ_s. Assume another value of θ_0 and find the corresponding θ_s.

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Repeat process until maximum value of θ_s is obtained. This corresponds to $\partial\theta_s/\partial\theta_o = 0$. These steps are repeated for other η_s. It is seen therefore that for any given η_s there are many values of θ_o (and therefore of θ_s) which satisfy the steady-state equation. The interest however is in the maximum θ_s which is still steady-state. This method was not used in the final numerical work because it was thought to be too time consuming.

A more efficient method has been developed and is now described. Choose an arbitrary θ_o and add a small perturbation $\varepsilon(\eta)$ ($\varepsilon \ll \theta$) to θ in Eq. (3) and obtain a differential equation in terms of θ , ε , and η . This new equation is solved simultaneously with Eq. (3). The value of η where ε vanishes is η_s, and the temperature there is θ_s. This is the point where $\partial\theta_s/\partial\theta_o = 0$. These steps are repeated for other θ_o . This method has the obvious advantage in that it is not an interative process and hence uses much less machine time than the previous method described. The mathematical analysis will now be described.

Replace θ by $\theta + \varepsilon$ in Eq. (3). This gives

$$\theta'' + \frac{m}{\eta}\theta' + \varepsilon'' + \frac{m}{\eta}\varepsilon' = -\exp\left[-\frac{1}{\theta\left(1+\frac{\varepsilon}{\theta}\right)}\right] \simeq -\exp\left[-\frac{1}{\theta}\left(1-\frac{\varepsilon}{\theta}\right)\right]$$

$$\simeq -\left(1+\frac{\varepsilon}{\theta^2}\right)\exp\left(-\frac{1}{\theta}\right) \tag{6}$$

Substituting Eq. (3) into Eq. (6) gives

$$\varepsilon'' + \frac{m}{\eta}\varepsilon' = -\frac{\varepsilon}{\theta^2}\exp\left(-\frac{1}{\theta}\right) \tag{7}$$

or

$$\varepsilon'' + \frac{m}{\eta}\varepsilon' = \frac{1}{\theta^2}\left(\theta'' + \frac{m}{\eta}\theta'\right)\varepsilon \tag{8}$$

To eliminate the exponential from Eq. (3), take the logarithmic derivative. This gives

$$\theta''' + \frac{m}{\eta}\theta'' - \frac{m}{\eta^2}\theta' = \frac{\theta'}{\theta^2}\left(\theta'' + \frac{m}{\eta}\theta'\right) \tag{9}$$

Eqs. (8) and (9) are solved simultaneously for θ and ε subject to the conditions

$$\begin{cases}\eta = \eta_s \\ \varepsilon = 0\end{cases} \qquad \begin{cases}\varepsilon = \varepsilon_o = \text{arbitrary constant} \\ \eta = 0\end{cases} \tag{10}$$

The point η_s and hence the temperature θ_s at this point are defined as those values of η and θ where $\varepsilon = 0$. The solution of Eqs. (8) and (9) is carried out by a finite difference technique.

The values of η_s, θ_s, and δ for different θ_o are tabulated in Tables I, II, and III. The parameter δ is easily evaluated by noting that

$$\delta = \left(\frac{\eta_s}{\theta_s}\right)^2 \exp\left(-\frac{1}{\theta_s}\right) \tag{11}$$

Frank-Kamenetzky and Chambré found that the critical values of δ for the slab δ_{sl}, the cylinder δ_{cyl}, and the sphere δ_{sph}, were

$$\delta_{sl}^* = 0.88, \qquad \delta_{cyl} = 2.00, \qquad \delta_{sph} = 3.32 \tag{12}$$

where it was assumed that $T - T_s \ll T_s$. Tables I, II, and III show that the Frank-Kamenetzky approximation is certainly more valid the greater the magnitude of b or the lower the surface temperature. The above values represent the asymptotic values of Tables I, II, and III.

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* A more precise value of δ with the same approximation is 0.87846 which was found by Enig (3).

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ANALYSIS OF RESULTS

The errors in η_s or θ_s which arise when the asymptotic values of δ are used instead of the more exact ones from Table I, II, and III, can be easily calculated. From Eq. (11) it is found that

$$\frac{d\delta}{\delta} = 2\frac{d\eta_s}{\eta_s} + \left(\frac{1}{\theta_s} - 2\right)\frac{d\theta_s}{\theta_s} \tag{13}$$

If $\eta_s\{\theta_s\}$ $\theta_s\{\eta_s\}$ is arbitrarily chosen then the error Δ in calculating due to an error $\Delta\delta$ in choice of δ is given by

$$\frac{\Delta\theta_s}{\theta_s} = \frac{\theta_s}{1-2\theta_s}\frac{\Delta\delta}{\delta} \tag{14}$$

$$\frac{\Delta\eta_s}{\eta_s} = \frac{1}{2}\frac{\Delta\delta}{\delta} \tag{15}$$

where θ_s, η_s, and δ are the values given in Tables I, II, and III, and $\Delta\delta \equiv \delta$ from Table minus asymptotic δ. From the Tables it is seen that $\Delta\delta$ and therefore $\frac{1}{\theta_s}\Delta\theta_s$ and $\frac{1}{\eta_s}\Delta\eta_s$ increase with increasing θ_s.

For all practical cases $\theta_s < 0.1$. Table IV gives the percentage error in calculating θ_s and η_s when the asymptotic values of δ are used instead of the exact ones from Tables I, II, and III.

For given θ_s $(\theta_s < 0.1)$, the greatest percentage error in calculating the critical thickness is less than 6%. For given η_s $(\eta_s > 10)$, the greatest percentage error in calculating the critical surface temperature is less than 1.5%. It should be noted that $\frac{1}{\theta_s}\Delta\theta_s$ and $\frac{1}{\eta_s}\Delta\eta_s$ decrease very markedly as θ_s decreases. In the range $\theta_s \sim 0.01$, the percentage errors are completely negligible. Hence the Frank-Kamentzky approximation is very good for all practical cases.

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ENGINEERING APPLICATIONS

The possibility of the self-ignition of explosives, propellants, or fuels that are kept in storage presents a hazard which must always be considered. Especially today, there has been a trend toward the design of large rocket propellant grains and it is important to know how large a grain may be under given temperature conditions before it presents a "spontaneous" ignition hazard.

With the aid of Tables I, II, and III, a sample calculation of the critical size of cordite propellant for various surface temperatures for different geometries is given in Table V. The numerical constants for cordite (4) are as follows:

$$E = 50,000 \text{ cal mole}^{-1} \quad \lambda = 5.3 \times 10^{-4} \text{cal cm}^{-1}\text{sec}^{-1}\text{deg}^{-1}$$

$$z = 10^{21.8} \text{ sec}^{-1} \quad \rho Q = 770 \text{ cal cm}^{-3}$$

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The authors wish to acknowledge the great help of Mr. L. D. Krider who programmed the simultaneous solution of Eqs. (8) and (9) on the 650 Magnetic Drum Data Processing Machine.

LIST OF REFERENCES

(1)   D. A. Frank-Kamenetsky, Acta Physicochimica U.R.S.S. 10, 365 (1939).

(2)   P. L. Chambré, J. Chem. Phys. 20, 1795 (1952).

(3)   J. W. Enig, NAVORD Report 4267 (1956).

(4)   D. M. Clemmow and J. D. Huffington, Royal Aircraft Establishment, Farnborough, Technical Note No: R.P.D. 128, Sept. 1955.

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TABLE I

VALUES OF η_s, θ_s, AND δ FOR DIFFERENT θ_0 FOR SLAB

θ_0	0.01 $LOG_{10}\ \eta_s$	θ_s	δ
0.500000000	0.003334277	0.244590484	1.202145442
0.333333333	0.003845221	0.213849282	1.197549012
0.250000000	0.004816405	0.181077142	1.120729228
0.200000000	0.006047844	0.155225235	1.071969426
0.166666667	0.007446704	0.135263884	1.039098934
0.142857143	0.008962328	0.119622854	1.015618961
0.125000000	0.010564037	0.107117486	0.998063561
0.111111111	0.012231847	0.096923984	0.984463231
0.100000000	0.013952009	0.088470961	0.973626084
0.090909091	0.015714680	0.081355594	0.964791793
0.083333333	0.017512487	0.075287963	0.957454843
0.076923077	0.019339916	0.070055022	0.951266276
0.071428571	0.021192599	0.065497137	0.945976180
0.066666667	0.023067081	0.061492562	0.941402615
0.062500000	0.024960571	0.057946931	0.937409392
0.058823529	0.026870783	0.054786049	0.933893604
0.055555556	0.028795819	0.051950803	0.930774338
0.052631579	0.030734086	0.049393541	0.927987873
0.050000000	0.032684237	0.047075438	0.925484184
0.047619048	0.034645117	0.044964551	0.923221482
0.045454545	0.036615736	0.043034363	0.921167275
0.043478261	0.038595233	0.041262686	0.919294372
0.041666667	0.040582854	0.039630009	0.917579076
0.040000000	0.042577941	0.038122847	0.916002167
0.038461538	0.044579913	0.036725231	0.914548626
0.037037037	0.046588248	0.035426299	0.913203151
0.035714286	0.048602490	0.034215973	0.911956034
0.034482759	0.050622223	0.033085498	0.910794084
0.033333333	0.052647081	0.032027237	0.909712792
0.032258065	0.054676722	0.031034493	0.908699944
0.031250000	0.056710850	0.030101372	0.907751763
0.030303030	0.058749187	0.029222665	0.906860742
0.029411765	0.060791482	0.028393754	0.906023372
0.028571429	0.062837509	0.027610526	0.905234718
0.027777778	0.064887056	0.026869309	0.904489649
0.027027027	0.066939933	0.026166814	0.903784735
0.026315789	0.068995963	0.025500087	0.903117798
0.025641026	0.071054978	0.024866467	0.902484812
0.025000000	0.073116834	0.024263549	0.901884662
0.024390244	0.075181387	0.023689156	0.901314175
0.023809524	0.077248508	0.023141311	0.900769146
0.023255814	0.079318078	0.022618218	0.900252013
0.022727273	0.081389981	0.022118236	0.899756311
0.022222222	0.083464119	0.021639868	0.899284487
0.021739130	0.085540390	0.021181743	0.898832985
0.021276596	0.087618699	0.020742604	0.898400948
0.020833333	0.089698969	0.020321293	0.897985868
0.020408163	0.091781111	0.019916749	0.897588339
0.020000000	0.093865052	0.019527990	0.897205861
0.019607843	0.095950726	0.019154111	0.896841924
0.019230769	0.098038059	0.018794273	0.896489488
0.018867925	0.100126987	0.018447701	0.896150191
0.018518519	0.102217459	0.018113674	0.895824430
0.018181818	0.104309417	0.017791523	0.895510746
0.017857143	0.106402799	0.017480627	0.895206449
0.017543860	0.108497561	0.017180406	0.894913999
0.017241379	0.110593658	0.016890320	0.894633387
0.016949153	0.112691033	0.016609864	0.894358836
0.016666667	0.114789655	0.016358587	0.894095897
0.016393443	0.116889479	0.016075987	0.893841268
0.016129032	0.118990470	0.015821711	0.893596382
0.015873016	0.121092579	0.015575352	0.893358353
0.015625000	0.123195782	0.015336544	0.893124704
0.015384615	0.125300043	0.015104947	0.892900779
0.015151515	0.127405322	0.014880240	0.892685750
0.014925373	0.129511598	0.014662118	0.892474676
0.014705882	0.131618841	0.014450297	0.892271870
0.014492754	0.133727009	0.014244508	0.892072193
0.014285714	0.135836100	0.014044496	0.891880363
0.014084507	0.137946065	0.013850022	0.891691038
0.013888889	0.140056892	0.013660859	0.891507913
0.013698630	0.142168556	0.013476793	0.891333239
0.013513514	0.144281022	0.013297620	0.891157982
0.013333333	0.146394297	0.013123147	0.890990764
0.013157895	0.148508325	0.012953194	0.890830549
0.012987013	0.150623111	0.012787584	0.890668518
0.012820513	0.152738624	0.012626156	0.890516769
0.012658228	0.154854849	0.012468751	0.890363200
0.012500000	0.156971771	0.012315222	0.890216628
0.012345679	0.159089367	0.012165427	0.890071515
0.012195122	0.161207622	0.012019232	0.889931751
0.012048193	0.163326521	0.011876509	0.889798566
0.011904762	0.165446052	0.011737134	0.889662329
0.011764706	0.167566193	0.011600993	0.889535741
0.011627907	0.169686937	0.011467972	0.889403229
0.011494253	0.171808263	0.011337968	0.889281999
0.011363636	0.173930171	0.011210877	0.889160583
0.011235955	0.176052625	0.011086604	0.889042048
0.011111111	0.178175631	0.010965055	0.888923734
0.010989011	0.180299171	0.010846143	0.888815668
0.010869565	0.182423239	0.010729781	0.888703319
0.010752688	0.184547814	0.010615889	0.888592211
0.010638298	0.186672885	0.010504390	0.888487664
0.010526316	0.188798450	0.010395208	0.888383128
0.010416667	0.190924495	0.010288272	0.888280446
0.010309278	0.193051025	0.010183513	0.888182275
0.010204082	0.195177989	0.010080867	0.888086568
0.010101010	0.197305426	0.009980268	0.887989850
0.010000000	0.199433296	0.009881657	0.887893551
0.005000000	0.413572317	0.004970372	0.883257842
0.001000000	2.143760855	0.000998814	0.879830274

TABLE II

VALUES OF η_s, θ_s, AND δ FOR DIFFERENT θ_0 FOR CYLINDER

θ_0	0.01 $\log_{10} \eta_s$	θ_s	δ
0.500000000	0.005245378	0.238114598	2.962237867
0.333333333	0.005850682	0.205990074	2.711200160
0.250000000	0.006872508	0.174976024	2.549921911
0.200000000	0.008135458	0.150611934	2.442559084
0.166666667	0.009555802	0.131712789	2.369332331
0.142857143	0.011087014	0.116824351	2.316550428
0.125000000	0.012700550	0.104862952	2.276815135
0.111111111	0.014377643	0.095072364	2.245867501
0.100000000	0.016105283	0.086924855	2.221102711
0.090909091	0.017874094	0.080046098	2.200845887
0.083333333	0.019677081	0.074165162	2.183974898
0.076923077	0.021508896	0.069081972	2.169709639
0.071428571	0.023365356	0.064645957	2.157490295
0.066666667	0.025243122	0.060741837	2.146908476
0.062500000	0.027139498	0.057279957	2.137656008
0.058823529	0.029052262	0.054189612	2.129498087
0.055555556	0.030979572	0.051414317	2.122250519
0.052631576	0.032919884	0.048908430	2.115771227
0.050000000	0.034872004	0.046634686	2.109936443
0.047619048	0.036834553	0.044562355	2.104665731
0.045454545	0.038806696	0.042665887	2.099875644
0.043478261	0.040787587	0.040923867	2.095505281
0.041666667	0.042776493	0.039318213	2.091501608
0.040000000	0.044772753	0.037833551	2.087818926
0.038461538	0.046776113	0.036456724	2.084420431
0.037037037	0.048785370	0.035176426	2.081273441
0.035714286	0.050800231	0.033982855	2.078365248
0.034482759	0.052820842	0.032867511	2.075648562
0.033333333	0.054846523	0.031822955	2.073114091
0.032258065	0.056876930	0.030842666	2.070742455
0.031250000	0.058911786	0.029920895	2.068520154
0.030303030	0.060950810	0.029052561	2.066433561
0.029411765	0.062993743	0.028233154	2.064472394
0.028571429	0.065040379	0.027458656	2.062620413
0.027777778	0.067090498	0.026725476	2.060874388
0.027027027	0.069143922	0.026030398	2.059219635
0.026315789	0.071200470	0.025370529	2.057655590
0.025641026	0.073259972	0.024743263	2.056171463
0.025000000	0.075322290	0.024146243	2.054760802
0.024390244	0.077387296	0.023577334	2.053420480
0.023809524	0.079454842	0.023034598	2.052141846
0.023255814	0.081524812	0.022516272	2.050925291
0.022727273	0.083597102	0.022020744	2.049763404
0.022222222	0.085671611	0.021546545	2.048654499
0.021739130	0.087748238	0.021092327	2.047590944
0.021276596	0.089826887	0.020656855	2.046574821
0.020833333	0.091907489	0.020238991	2.045602677
0.020408163	0.093989946	0.019837689	2.044668154
0.020000000	0.096074193	0.019451986	2.043771813
0.019607843	0.098160153	0.019080987	2.042909990
0.019230769	0.100247760	0.018723869	2.042082348
0.018867925	0.102336964	0.018379869	2.041285905
0.018518519	0.104427691	0.018048275	2.040518308
0.018181818	0.106519903	0.017728428	2.039778352
0.017857143	0.108613536	0.017419718	2.039066420
0.017543860	0.110708527	0.017121570	2.038379652
0.017241379	0.112804847	0.016833453	2.037715949
0.016949153	0.114902443	0.016554870	2.037075058
0.016666667	0.117001275	0.016285354	2.036455279
0.016393443	0.119101310	0.016024470	2.035855355
0.016129032	0.121202501	0.015771810	2.035276056
0.015873016	0.123304793	0.015526992	2.034714001
0.015625000	0.125408183	0.015289656	2.034170215
0.015384615	0.127512632	0.015059463	2.033644918
0.015151515	0.129618092	0.014836099	2.033131954
0.014925373	0.131724527	0.014619261	2.032636588
0.014705882	0.133831939	0.014408669	2.032155054
0.014492754	0.135940276	0.014204057	2.031688041
0.014285714	0.138049518	0.014005172	2.031233711
0.014084507	0.140159637	0.013811780	2.030794576
0.013888889	0.142270604	0.013623654	2.030364754
0.013698630	0.144382441	0.013440583	2.029943629
0.013513514	0.146495024	0.013262367	2.029539599
0.013333333	0.148608454	0.013088812	2.029147563
0.013157895	0.150722607	0.012919742	2.028761835
0.012987013	0.152837528	0.012754983	2.028388618
0.012820513	0.154953164	0.012594372	2.028023270
0.012658228	0.157069520	0.012437755	2.027665293
0.012500000	0.159186566	0.012284984	2.027320644
0.012345679	0.161304270	0.012135920	2.026982567
0.010638298	0.188889121	0.010482389	2.023243617
0.010526316	0.191014779	0.010373662	2.022997960
0.010416667	0.193140912	0.010267167	2.022761161
0.010309278	0.195267536	0.010162835	2.022523238
0.010204082	0.197394561	0.010060604	2.022290862
0.010101010	0.199522118	0.009960407	2.022070489
0.010000000	0.201650061	0.009862187	2.021843894
0.005000000	0.415793170	0.004965445	2.010909415
0.003333333	0.631181622	0.003317560	2.007266211
0.002500000	0.847080488	0.002491348	2.005424564
0.002000000	1.063259154	0.001994461	2.004425537
0.001666667	1.279614305	0.001662820	2.003679650
0.001428571	1.496093375	0.001425744	2.003115420
0.001250000	1.712660411	0.001247836	2.002788992
0.001111111	1.929296117	0.001109401	2.002388158
0.001000000	2.145985449	0.000998615	2.002286485

TABLE III
VALUES OF η_S, θ_S, AND ϑ FOR DIFFERENT θ_0 FOR SPHERE

θ_0	$0.01 \log_{10} \eta_S$	θ_S	ϑ
0.500000000	0.006487343	0.231030422	4.901704114
0.333333333	0.007184535	0.198389833	4.495769704
0.250000000	0.008259503	0.169000299	4.230314300
0.200000000	0.009556939	0.146017414	4.058316234
0.166666667	0.011001440	0.128126758	3.939776484
0.142857143	0.012550503	0.113966900	3.853613914
0.125000000	0.014177762	0.102540453	3.788317446
0.111111111	0.015865734	0.093151122	3.737196257
0.100000000	0.017602209	0.085311039	3.696110864
0.090909091	0.019378336	0.078672406	3.662389597
0.083333333	0.021187476	0.072982308	3.634222718
0.076923077	0.023024551	0.068053135	3.610346528
0.071428571	0.024885538	0.063743121	3.589851736
0.066666667	0.026767259	0.059943339	3.572072095
0.062500000	0.028667121	0.056568799	3.556498963
0.058823529	0.030582970	0.053552273	3.542751457
0.055555556	0.032513028	0.050839922	3.530521738
0.052631579	0.034455819	0.048388127	3.519575175
0.050000000	0.036410042	0.046161199	3.509717740
0.047619048	0.038374615	0.044129659	3.500797048
0.045454545	0.040348603	0.042268941	3.492684447
0.043478261	0.042331194	0.040558432	3.485273869
0.041666667	0.044321639	0.038980686	3.478481091
0.040000000	0.046319349	0.037520856	3.472230407
0.038461538	0.048323734	0.036166229	3.466460743
0.037037037	0.050334310	0.034905847	3.461114744
0.035714286	0.052350650	0.033730221	3.456152817
0.034482759	0.054372322	0.032631094	3.451531536
0.033333333	0.056398991	0.031601243	3.447218790
0.032258065	0.058430323	0.030634331	3.443180195
0.031250000	0.060466075	0.029724762	3.439396121
0.030303030	0.062505907	0.028867591	3.435842660
0.029411765	0.064549607	0.028058421	3.432497975
0.028571429	0.066596985	0.027293334	3.429340066
0.027777778	0.068647812	0.026568826	3.426360999
0.027027027	0.070701873	0.025881754	3.423540024
0.026315789	0.072759054	0.025229294	3.420869354
0.025641026	0.074819139	0.024608898	3.418337877
0.025000000	0.076882045	0.024018257	3.415921708
0.024390244	0.078947546	0.023455285	3.413637405
0.023809524	0.081015610	0.022918082	3.411458521
0.023255814	0.083086082	0.022404921	3.409379003
0.022727273	0.085158846	0.021914224	3.407393913
0.022222222	0.087233794	0.021444547	3.405493557
0.021739130	0.089310830	0.020994569	3.403681338
0.021276596	0.091389881	0.020563079	3.401942061
0.020833333	0.093470884	0.020148958	3.400277422
0.020408163	0.095553701	0.019751180	3.398682692
0.020000000	0.097638342	0.019368797	3.397147289
0.019607843	0.099724645	0.019000932	3.395671396
0.019230769	0.101812579	0.018646774	3.394259487
0.018867925	0.103902082	0.018305572	3.392895544
0.018518519	0.105993133	0.017976627	3.391581359
0.018181818	0.108085624	0.017659290	3.390318412
0.017857143	0.110179524	0.017352960	3.389095212
0.017543860	0.112274854	0.017057072	3.387919323
0.017241379	0.114371438	0.016771101	3.386784626
0.016949153	0.116469256	0.016494559	3.385685306
0.016666667	0.118568366	0.016226986	3.384623224
0.016393443	0.120668611	0.015967953	3.383595197
0.016129032	0.122770067	0.015717056	3.382606579
0.015873016	0.124872548	0.015473921	3.381638168
0.015625000	0.126976191	0.015238191	3.380711679
0.015384615	0.129080828	0.015009533	3.379803558
0.015151515	0.131186475	0.014787635	3.378929884
0.014925373	0.133293159	0.014572200	3.378081296
0.014705882	0.135400784	0.014362951	3.377260360
0.014492754	0.137509262	0.014159626	3.376454136
0.014285714	0.139618699	0.013961974	3.375685829
0.014084507	0.141729030	0.013769763	3.374918202
0.013888889	0.143840185	0.013582772	3.374184992
0.013698630	0.145952116	0.013400790	3.373468272
0.013513514	0.148064877	0.013223620	3.372768016
0.013333333	0.150178499	0.013051071	3.372096030
0.013157895	0.152292817	0.012882968	3.371439713
0.012987013	0.154407830	0.012719138	3.370797186
0.012820513	0.156523627	0.012559423	3.370167427
0.012658228	0.158640124	0.012403668	3.369555797
0.012500000	0.160757339	0.012251728	3.368962762
0.012345679	0.162875200	0.012103464	3.368385437
0.012195122	0.164993691	0.011958746	3.367826464
0.012048193	0.167112820	0.011817448	3.367272422
0.011904762	0.169232590	0.011679448	3.366730410
0.011764706	0.171352975	0.011544634	3.366204977
0.011627907	0.173473940	0.011412896	3.365687031
0.011494253	0.175595514	0.011284130	3.365185620
0.011363636	0.177717664	0.011158237	3.364696252
0.011235955	0.179840310	0.011035122	3.364222257
0.011111111	0.181963494	0.010914693	3.363748468
0.010989011	0.184087247	0.010796865	3.363297794
0.010869565	0.186211511	0.010681553	3.362842784
0.010752688	0.188336340	0.010568677	3.362400712
0.010638298	0.190461551	0.010458162	3.361960969
0.010526316	0.192587351	0.010349935	3.361556549
0.010416667	0.194713531	0.010243924	3.361139103
0.010309278	0.196840295	0.010140061	3.360739410
0.010204082	0.198967473	0.010038285	3.360338392
0.010101010	0.201095021	0.009938530	3.359956054
0.010000000	0.203223067	0.009840738	3.359574512
0.005000000	0.417370688	0.004960000	3.340172579
0.003333333	0.632760447	0.003315528	3.334524173
0.002500000	0.848659885	0.002489977	3.331334024
0.002000000	1.064837900	0.001993582	3.329563694
0.001666667	1.281194936	0.001662209	3.328254479
0.001428571	1.497672783	0.001425295	3.327774849
0.001250000	1.714238590	0.001247491	3.326556165
0.001111111	1.930874744	0.001109129	3.326276109
0.001000000	2.147565469	0.000998394	3.325902353

TABLE IV

PERCENT ERROR IN θ_s AND η_s DUE TO ERROR IN δ

	θ_s	δ	$\Delta\delta$	$\dfrac{\Delta\delta}{\delta}$	$100\dfrac{\Delta\theta_s}{\theta_s}$	$100\dfrac{\Delta\eta_s}{\eta_s}$
Slab	0.1	0.9885	0.1100	0.1113	1.391	5.565
Cylinder	0.1	2.2604	2.2604	0.1152	1.440	5.760
Sphere	0.1	3.774	0.454	0.1203	1.504	6.015

TABLE V

CRITICAL VALUES OF b FOR DIFFERENT T_s FOR A SLAB, CYLINDER, AND SPHERE OF CORDITE

$T_s(^\circ K)$	b (cm)	Geometry
306	16,620	slab
306	25,250	cylinder
306	32,400	sphere
353	70.8	slab
353	107.2	cylinder
353	134.6	sphere
404	0.91	slab
404	1.41	cylinder
404	1.83	sphere

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